THE JOY OF X by Steven Strogatz, Cornell University NOTES!**1** |Page

Algebra deals with “relationships”

Geometry and Trigonometry deal with “shapes”.

Calculus is the mathematics of “change”. It domesticated “infinity”.

The derivative tells you how fast something is changing.

The integral tells you how much it’s accumulating.

Vector Calculus describes the invisible fields all around us. An example is the “vector field” around a magnet. Maxwell used this to determine the speed of light.

Theoretical Physics is based on finding the right differential equations and then solving them.

Probability and Statistics deal with ”data”.

Power Laws: See <http://wwwpersonal.umich.edu/~mejn/courses/2006/cmplxsys899/powerlaws.pdf>

Complex numbers are better than real numbers as they all have roots. They can be visualized as all numbers in the plane, with only the “real” numbers on the x axis. Approximating the three roots of a third degree equation results in fractal results at the boundaries.

Definitions help make math work.

The fundamental Theorem of Algebra does not work if 1 is defined as a prime.

(-a) x (-b) = +ab works for positive and negative numbers.

The eminent linguistic philosopher J L Austin of Oxford asserted in a lecture that there are many languages in which a double negative makes a positive, but none in which a double positive makes a negative. The Columbian philosopher Sidney Morgenbesser, sarcastically replied, “Yeah, yeah”.

“i”, the square root of minus 1, when squared equals minus 1.

An integer is a whole number that can be either greater than 0, called positive, or less than 0, called

negative.  **Zero** is neither **positive** nor **negative**.

**Zero factorial 0! = 1** for reasons that are similar to why x^0 = 1. Both are defined that way. But there

are reasons for these definitions; they are not arbitrary.

You cannot reason that x^0 = 1 by thinking of the meaning of powers as "repeated multiplications"

because you cannot multiply x zero times. Similarly, you cannot reason out 0! just in terms of the

meaning of factorial because you cannot multiply all the numbers from zero down to 1 to get 1.

Mathematicians \*define\* x^0 = 1 in order to make the laws of exponents work even when the

exponents can no longer be thought of as repeated multiplication. For example, (x^3)(x^5) = x^8

because you can add exponents. In the same way (x^0)(x^2) should be equal to x^2 by

adding exponents. But that means that x^0 must be 1 because when you multiply x^2 by it, the result

is still x^2. Only x^0 = 1 makes sense here.

In the same way, when thinking about combinations we can derive a formula for "the number of ways of choosing k things from a collection of n things." The formula to count out such problems is n!/k!(n-k)!.

For example, the number of handshakes that occur when everybody in a group of 5 people shakes hands can be computed using n = 5 (five people) and k = 2 (2 people per handshake) in this formula. (So the answer is 5!/(2! 3!) = 10).

Now suppose that there are 2 people and "everybody shakes hands with everybody else." Obviously there is only one handshake. But what happens if we put n = 2 (2 people) and k = 2 (2 people per handshake) in the formula? We get 2! / (2! 0!). This is 2/(2 x), where x is the value of 0!. The fraction reduces to 1/x, which must equal 1 since there is only 1 handshake. The only value of 0! that makes sense here is 0! = 1. And so we define 0! = 1.

Commutative law doesn’t always work in practice. The physicist Murry Gell-Mann wanted to go to an Ivy League grad school. He finally accepted an offer from MIT. He said he contemplated suicide but realized that attending MIT and committing suicide did not commute. He could always go to MIT and cooit suicide later, but not the other way around. As a physicist he also knew at the deepest level, nature disobeys the commutative law. It is why matter is solid and why atoms do not implode. Similarly there would otherwise be no Heisenberg Uncertainty Principle and atoms would collapse and nothing would exist.

In recording numbers as four vertical lines and then a line thru the four vertical lines to denote 5 … may represent a thumb across four fingers.

Brittany Gallivan, a junior in High School in 2002 determined the maximum number of times a paper can be folded in one direction. Every time you fold it, you double its thickness. Her correct answer is the formula:

L = Tpi/6 x (4 + 2 to the n) x (-1 = 2 to the n) where T = thickness L = length n = number of times.

In January 2002 she used a roll of toilet paper nearly ¾ miles long and after 7 hours had smashed the world record by folding it 12 times.

The Pythagorean Theorem says that space is flat, as opposed to curved. Einstein said no, that in his theory of Relativity, that gravity is not a force but a manifestation of the curvature of space. Rieman’s Non Euclidean Geometry implied the same thing.

An ellipse is the locus of points related to two fixed points. The sum of the distances from any point on the ellipse to the fixed points is the same. Consider an elliptical pool table. If there are two pockets, one at each focus, a ball sitting on one focus, when struck, will carom off the cushion and go into the other pocket … no matter where it strikes the cushion. (Forget thee trivial case of striking the closest cushion on the x axis.)

A parabola is the locus of points equidistant from a fixed point and a fixed line. The focusing property of a parabola is also of interest. Place a light bulb at the focus inside a reflector. Each ray will all be reflected in the direction perpendicular to the line. If one wants a suntan, lay at the focus and let the sun ‘s rays be reflected off a parabolic mirror and they will all be reflected to where you are at the focus. Do you like parabolas? See <http://www.youtube.com/watch?v=VSUKNxVXE4E> and <http://www.youtube.com/watch?v=rdSgqHuI-mw> and <https://mysite.du.edu/~jcalvert/>

The sections of a cone (ellipses, parabolas, and hyperbolas0 arise in calculus as trajectories of objects tugged by gravity.

If holes in a music box tape are punched on a Moebius Strip, it plays the music correctly, but the second time through, plays the notes upside down. A Moebius Strip conveyor belt lasts twice as long as a regular one as both sides are used.

Infinity is neither even or odd.

Read “All about e” at <http://www.maa.org/sites/default/files/pdf/upload_library/22/Chauvenet/mccartin.pdf>